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Неопределени интеграли

$f(x)$ (a, b)

$F(x)$ - примитивна функция за f

$F'(x) = f(x)$

$$\left(\int f(x)\,dx\right)' = f(x)$$

неопределен интеграл

1) $\int \sin x\,dx = -\cos x$

2) $\int \cos x\,dx = \sin x$

3) $\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1}$ $\alpha \neq -1$

4) при $\alpha = -1 \Rightarrow \int \frac{dx}{x} = \ln(x)$

при $x > 0$ $\ln|x| = \ln x$

$(\ln x)' = 1/x$

при $x < 0$ $\ln|x| = \ln(-x)$

$(\ln(-x))' = 1/x = \frac{1}{-x}(-1)$

5) $\int \frac{dx}{\cos^2 x} = \operatorname{tg} x$

6) $\int \frac{dx}{\sin^2 x} = -\operatorname{cotg} x$

7) $\int \frac{dx}{1+x^2} = \operatorname{arc\,tg} x$

8) $\int \frac{dx}{\sqrt{1-x^2}} = \operatorname{arc\,sin} x$

9) $\int \frac{dx}{\sqrt{1-x^2}} = -\operatorname{arc\,cos} x$

10) $\int e^x\,dx = e^x$

$$\int e^{ax}\,dx = \frac{a}{a}\int e^{ax}\,dx = \frac{1}{a}\int a e^{ax}\,dx =$$

$$= \frac{1}{a}\int e^{ax}\,d(ax) = \int \frac{e^{ax}}{a}\,d(ax)$$

$$\frac{d}{dx}\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x)$$

$$\int \left(f'(x)g(x) + f(x)g'(x)\right)dx = f(x)g(x)$$

$$\int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx = f(x)g(x)$$

$$\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

но $dg(x) = g'(x)\,dx \Rightarrow$

$$\int g(x)\,df(x) = f(x)g(x) - \int f(x)\,dg(x)$$

интегрирование по части

Пример

$$\int \ln x\,dx = \frac{x\ln x - \int x\,d\ln x}{2} =$$

$$= \frac{x\ln x - \int dx}{2} = x\ln x - x$$

$\int dx = x \qquad d e^x = e^x dx$

$I_1 = \int e^x \sin x\,dx$; $I_2 = \int e^x \cos x\,dx$

Разглеждаме

$$I_1 = \int \sin x\,de^x = e^x \sin x - \underbrace{\int e^x \cos x\,dx}_{I_2}$$

$$I_2 = \int \cos x\,de^x = e^x \cos x + \underbrace{\int e^x \sin x\,dx}_{I_1}$$

$$I_1 = e^x \sin x - e^x \cos x - I_1 = \frac{1}{2} e^x (\sin x - \cos x) + C$$

$$I_2 = e^x \cos x + \frac{1}{2} e^x (\sin x - \cos x) =$$

$$\frac{e^x \cos x}{2} + \frac{1}{2} e^x \sin x = \frac{1}{2} e^x (\cos x + \sin x)$$

$$\int R(x)\,dx$$

$$\frac{P(x)}{Q(x)} \Rightarrow Q(x) = A(x-a_1)^{\alpha_1}(x-a_2)^{\alpha_2}\cdots(x-a_\tau)^{\alpha_\tau} \cdot (x-b_1)^{\beta_1}(x-c_1)^{\beta_1}\cdot(x-b_2)^{\beta_2}(x-c_2)^{\beta_2}\cdots(x-b_s)^{\beta_s}(x-c_s)^{\beta_s}$$

$$\frac{A_1 a_1}{(x-a_1)^{\alpha_1}} + \frac{A_2 a_1}{(x-a_1)^{\alpha_1-1}} + \cdots + \frac{A_{\alpha_1} a_1}{x-a_1}$$

$$\frac{M_1 b_1 + N_1 c_1}{(x-b_1)^{\beta_1}(x-c_1)^{\beta_1}}$$